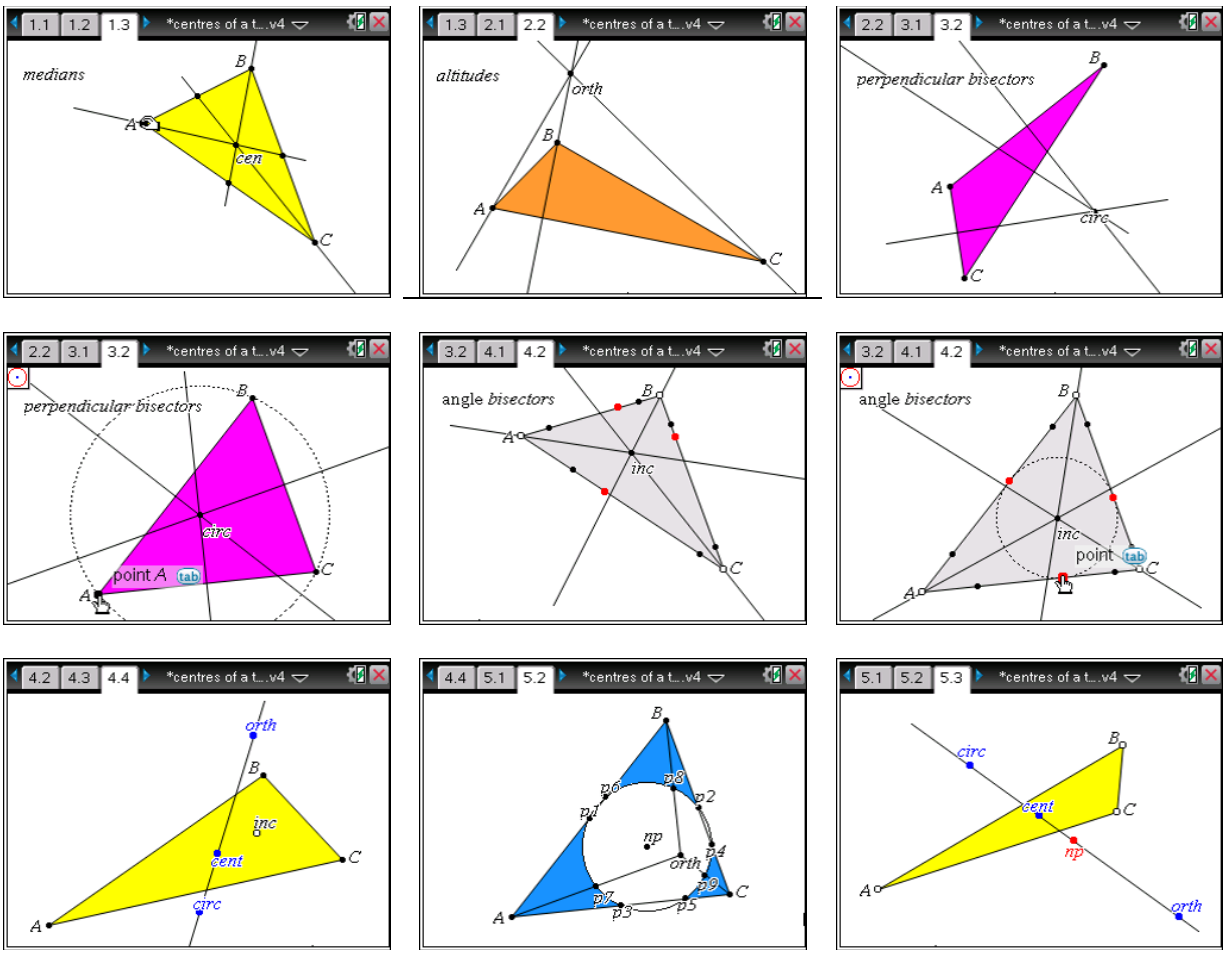
Centres of a Triangle

Teacher Notes

Introduction

The aim of this activity is to investigate some of the centres of a triangle and to discover the Euler Line. The activity enables students to find the centroid, orthocentre, circumcentre and intersection of the angle bisectors and to discover that three of these centres lie on a line. The activity then extends to finding the Nine-point Centre and discovering that this centre also lies on the line, which is revealed to be the Euler Line.
During the activity students also draw the circumcircle and incircle and the ratio the centroid divides each median is investigated at the end.



Resources

TI-Nspire document Centres of a Triangle and a worksheet that supports the activity.

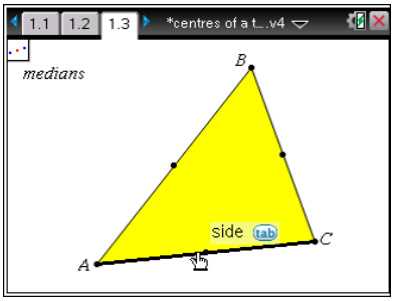
Skills required

Students need to be able to use the construction menus to draw medians, altitudes and perpendicular bisectors (instructions given on the worksheet).

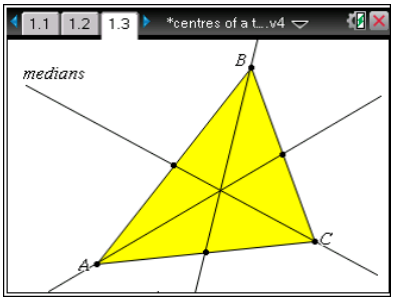
The activity

1). Students construct the 3 **medians** of the triangle ABC.

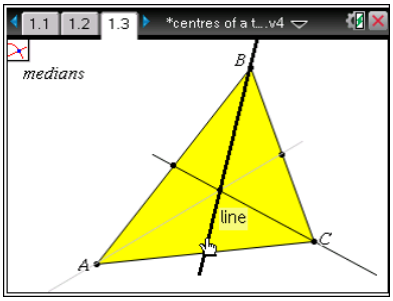
Find the **midpoint** of each line and draw the **medians.**



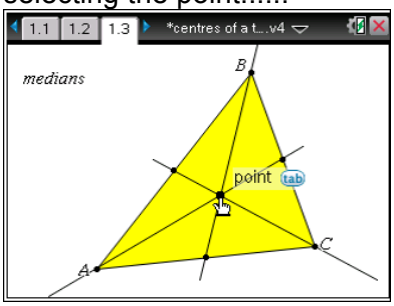
Draw these lines on the worksheet.



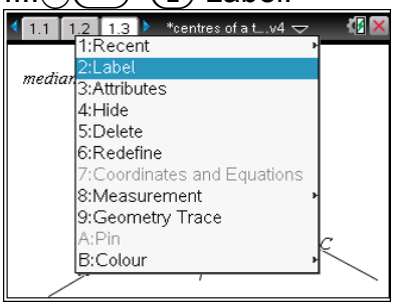
Find the **point of intersection**.



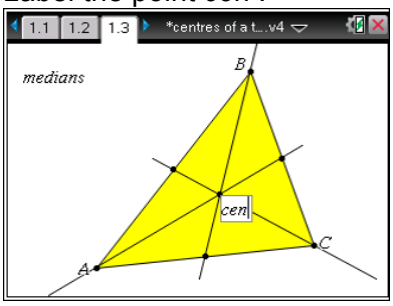
Label the centroid by selecting the point.....



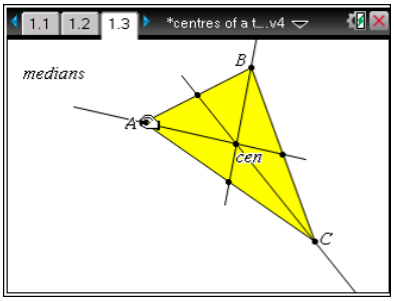
....ctrl menu (2) Label.

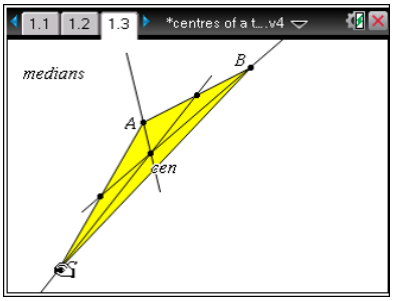


Label the point *cen* .



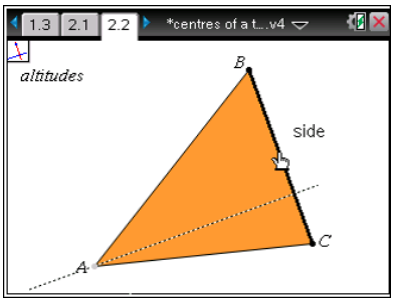
Grab A, B and C and note that the medians remain concurrent inside the triangle.



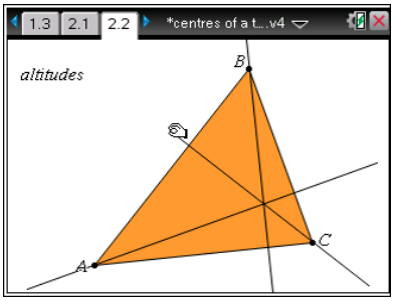


2). Students construct the 3 **altitudes** of the triangle ABC.

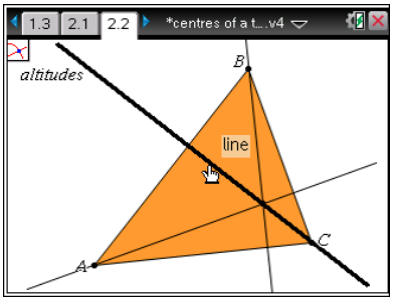
Draw the **altitudes** from A, B and C.



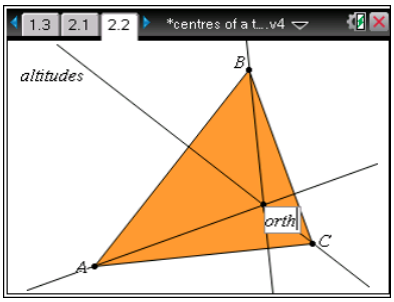
Extend the altitudes on the handheld and draw them on the worksheet.



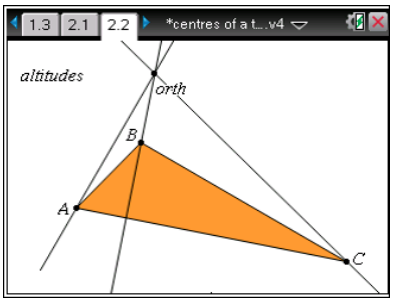
Find the **point of intersection**



Label the orthocentre *orth*.

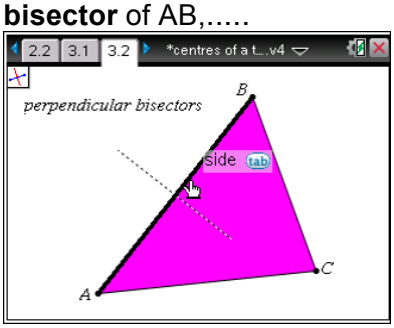


Grab A, B and C and note that the altitudes remain concurrent and that the intersection point can lie outside the triangle.

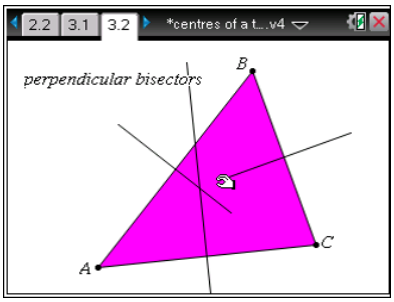


3). Students construct the 3 **perpendicular bisectors** of the triangle ABC.

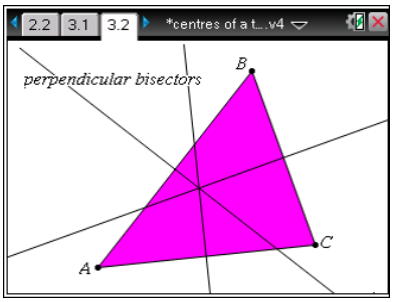
Draw the **perpendicular bisector** of AB,.....



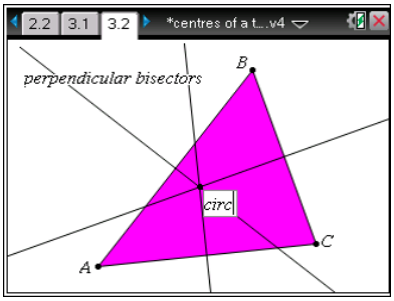
....AC and BC. Grab the end of the lines and extend them.



Draw the lines on the worksheet.

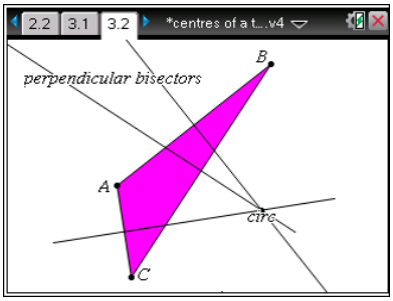


Find the **point of intersection** and label the circumcentre *circ*.

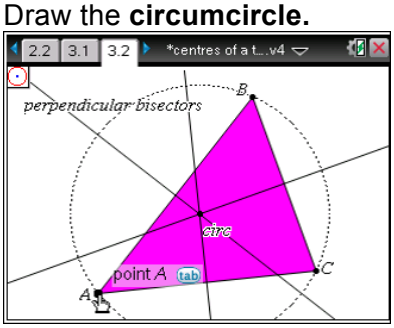


Grab A, B and C and note that the perpendicular bisectors remain concurrent and that the intersection point can lie outside the triangle.

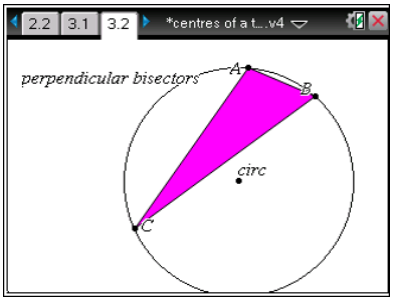
Enter ctrl esc until the screen returns to the one shown on the left.



Draw the **circumcircle.**

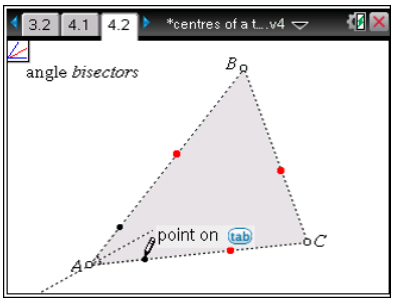


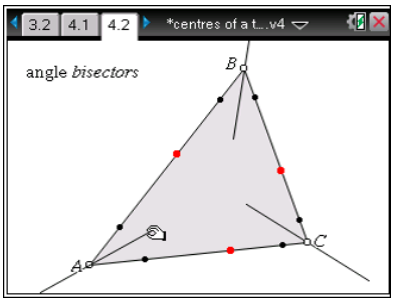
Hide the perpendicular bisectors.
Grab A, B and C and note that the circumcircle continues to pass trough A, B and C.

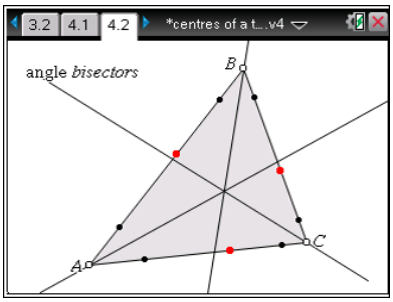


4). Students construct the 3 **angle bisectors** of the triangle ABC.

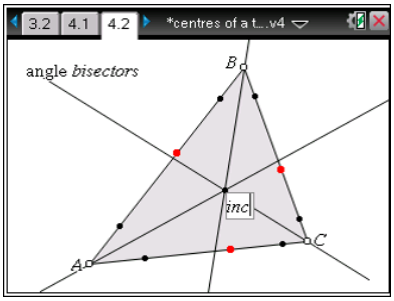
Draw the bisector of angle A,. ..B and C. Grab the end of the lines and extend them. Draw the lines on the worksheet.





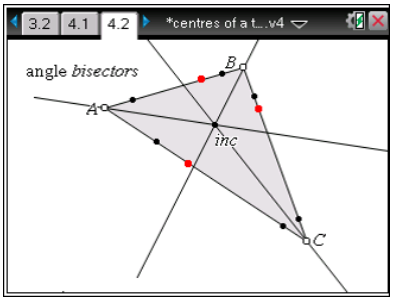


Find the **point of intersection** and label the incentre *inc.*

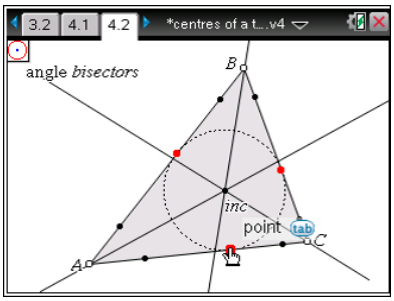


Grab A, B and C and note that the angle bisectors remain concurrent and that the intersection point lies inside the triangle.

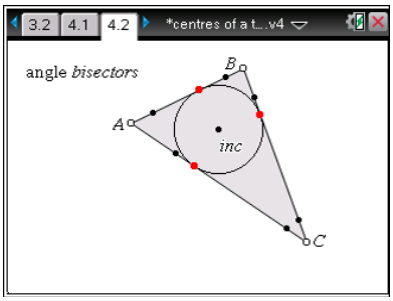
enter ctrl esc until the screen returns to the one shown on the left.

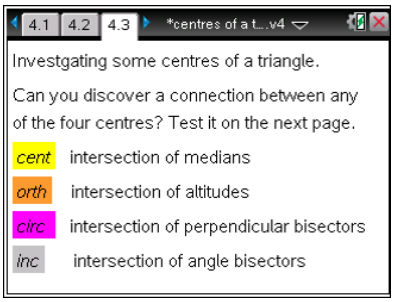


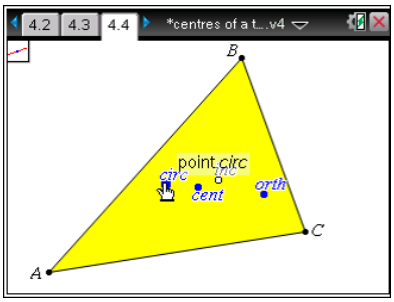
Draw the **incircle**.

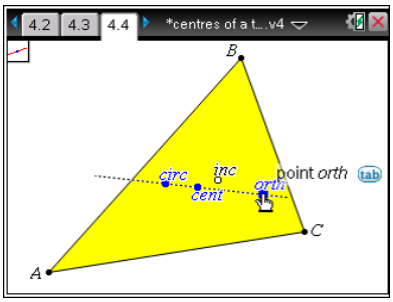


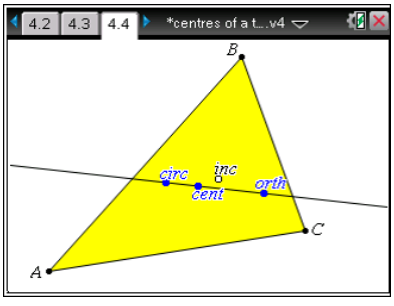
Hide the angle bisectors. Grab A, B and C and note that the largest circle inside the triangle is the incircle.





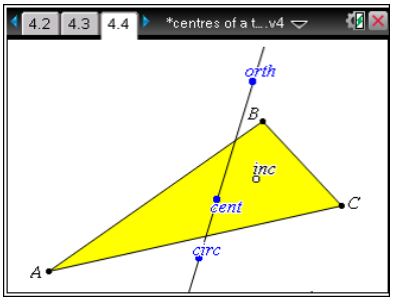






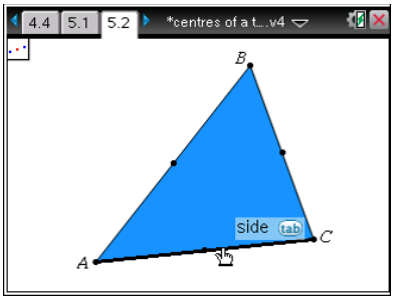
Grab A,B and C to see which centres remain inside the triangle.

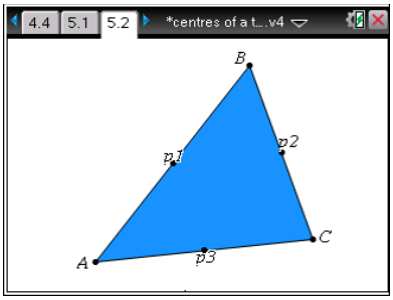
The 4 centres become the same point when triangle ABC is equilateral.



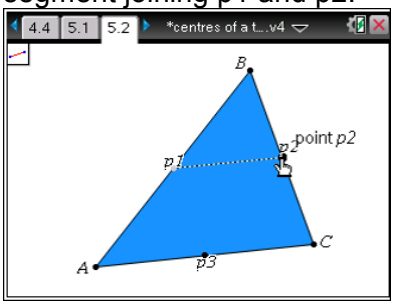
5). Students construct the **NINE-POINT CENTRE** of the triangle ABC.

The first 3 points are the midpoints of AB, BC and AC and are labelled p1, p2 and p3.

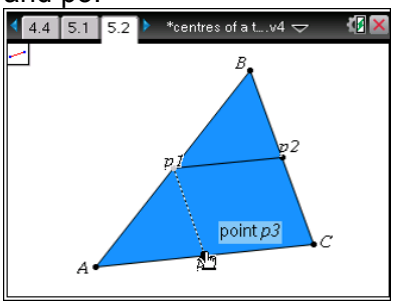




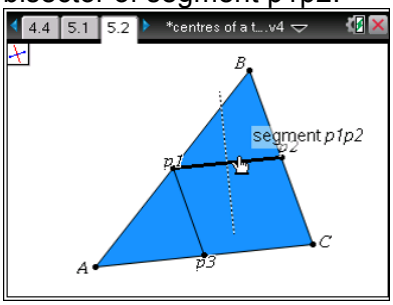
To find the centre draw the segment joining p1 and p2.



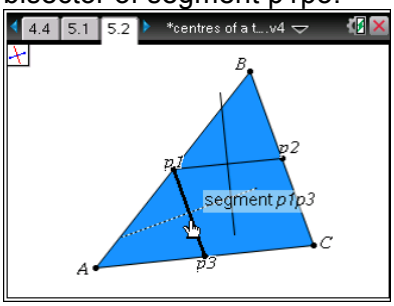
Draw the segment joining p1 and p3.



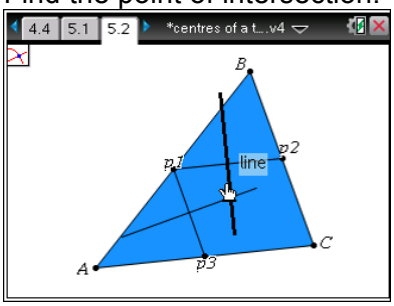
Find the perpendicular bisector of segment p1p2.



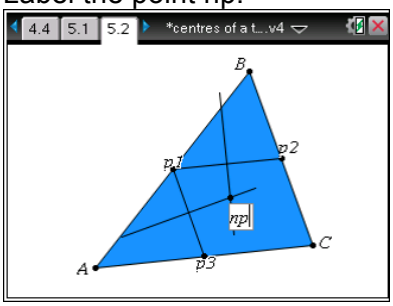
Find the perpendicular bisector of segment p1p3.



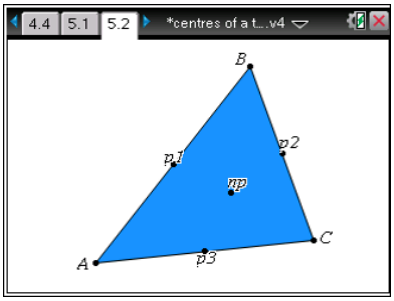
Find the point of intersection.



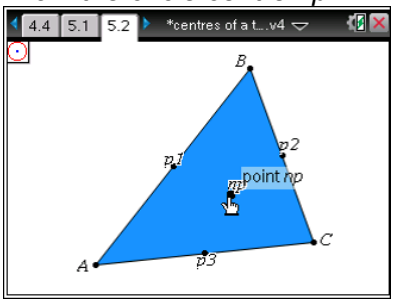
Label the point np.



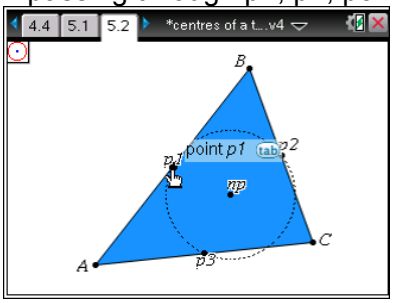
Hide the construction lines.



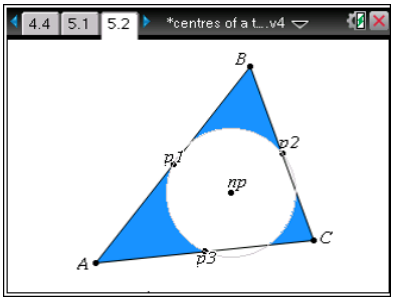
Draw the circle centre *np*.....



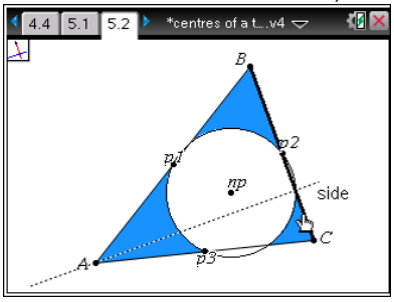
...passing through p1, p2, p3.



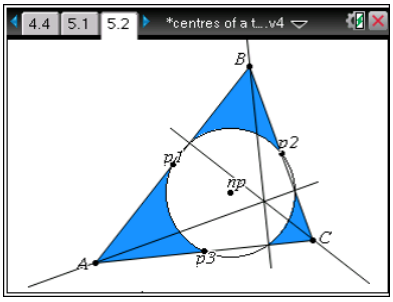
Fill the circle with white.



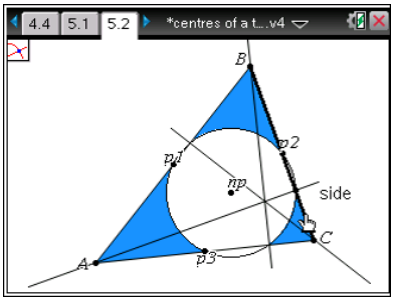
Draw the altitudes from A,...



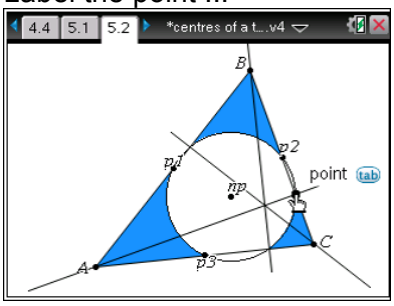
...B and C.



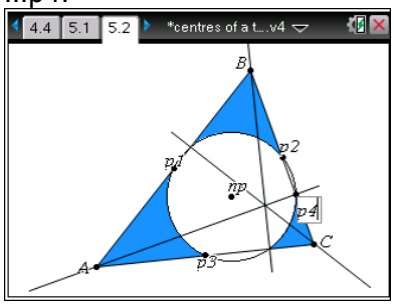
Find the intersection of the altitude from A with BC.



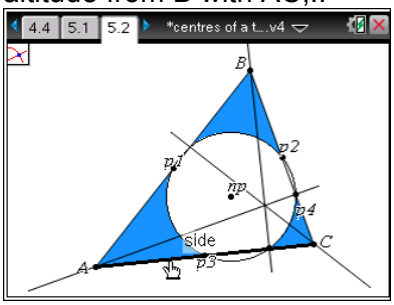
Label the point ...



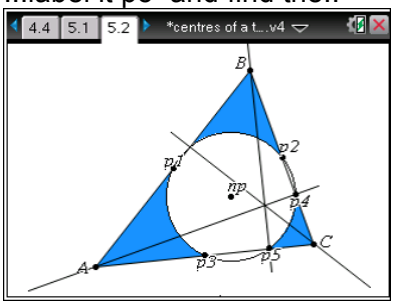
...p4.



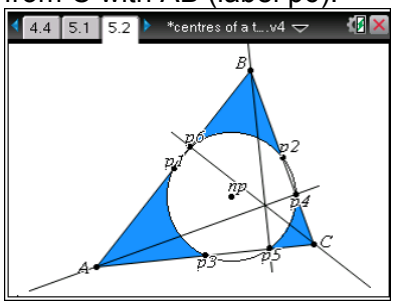
Find the intersection of the altitude from B with AC,..



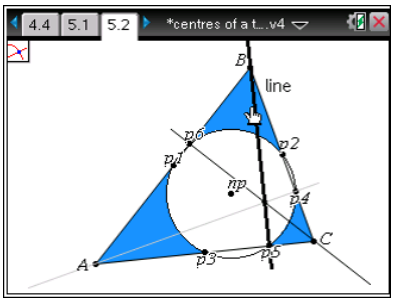
...label it p5 and find the..



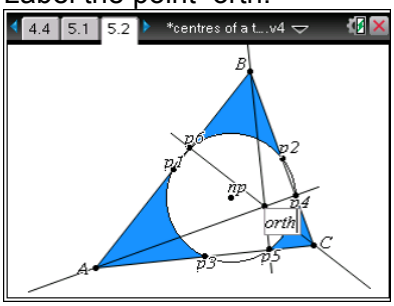
.. intersection of the altitude from C with AB (label p6).



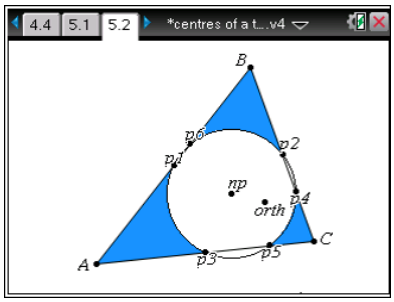
Find the intersection of the altitudes from A and B.



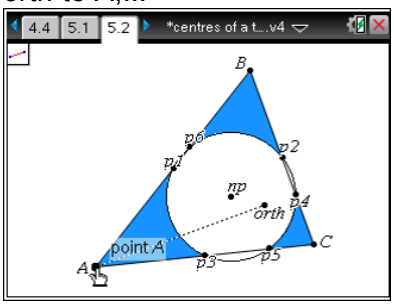
Label the point orth.



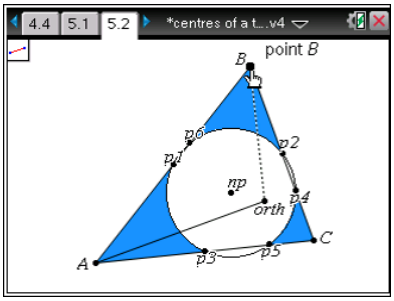
Hide the altitudes.



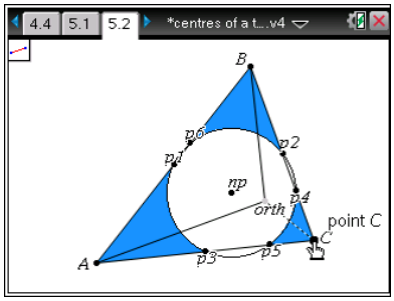
Draw the segment from *orth* to A,...



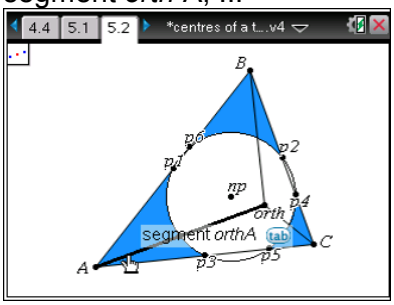
...*orth* to B and...



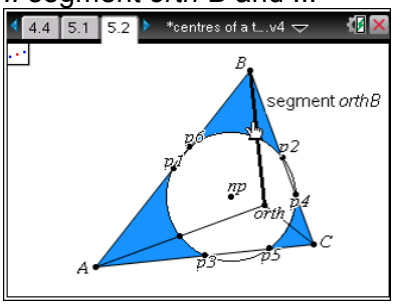
.....*orth* to C.



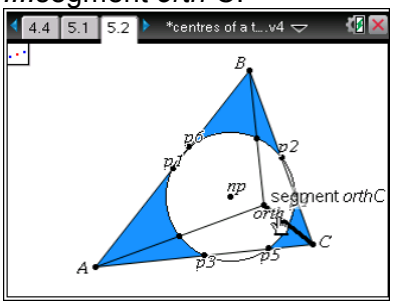
Find the midpoints of segment *orth* A, ...



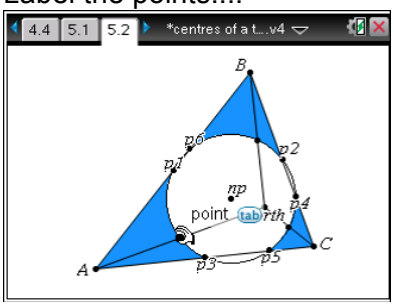
.. segment *orth* B and ...



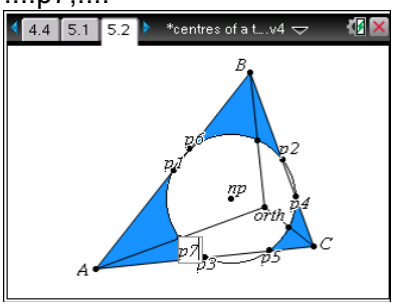
....segment *orth* C.



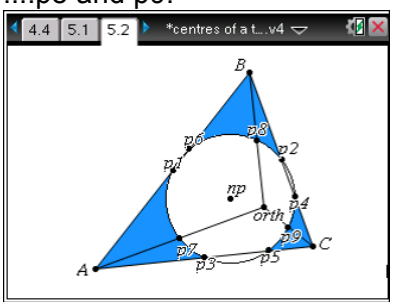
Label the points....



....p7,....

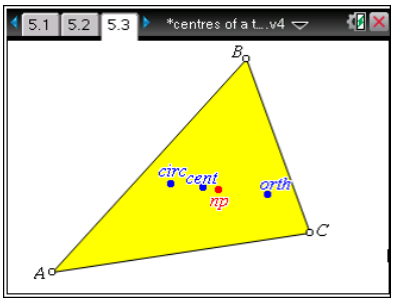


....p8 and p9.

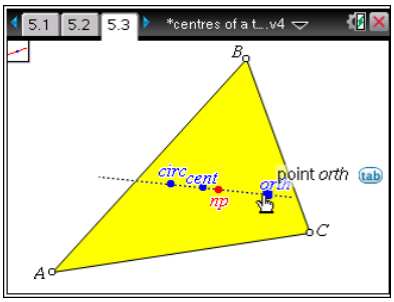


The nine points p1 to p9 lie on the circumference of the circle with centre np.

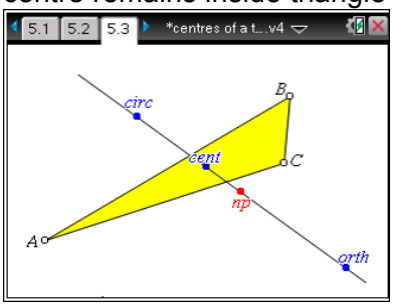
Find the connection between the four centres.



Draw a line between *circ* and *orth*.



Grab A,B and C to see which centre remains inside triangle



Use the information on the screen to complete the worksheet.