OCR Further Pure 1

Complex Numbers

Section 1: Introduction to complex numbers

Multiple Choice Test

1) $(3 + 4i) - (2 - i) =$

(a) $5 + 3i$
(b) $5 + 5i$
(c) $1 + 5i$
(d) $1 + 3i$
(e) I don't know

2) $(2 + i)(3 - 2i) =$

(a) $4 + 5i$
(b) $8 + 5i$
(c) $4 - i$
(d) $8 - i$
(e) I don't know

3) $[(1 - 2i) + (1 + i)](-3 + i) =$

(a) $5 - 5i$
(b) $-7 - i$
(c) $-7 + 5i$
(d) $-5 + 5i$
(e) I don't know

4) The roots of the equation

$$z^2 + 6z + 10 = 0$$

are

(a) $-3 + i, -3 - i$
(b) $3 + 2i, 3 - 2i$
(c) $-3 + 2i, -3 - 2i$
(d) $3 + i, 3 - i$
(f) I don't know

5) Given that $p + q\mathrm{i} = \frac{1}{12 - 5\mathrm{i}}$, the values of p and q are given by

(a) $p = \frac{12}{119}, q = \frac{5}{119}$
(b) $p = \frac{12}{169}, q = -\frac{5}{169}$
(c) $p = \frac{12}{169}, q = \frac{5}{169}$
(d) $p = \frac{12}{119}, q = -\frac{5}{119}$
(e) I don't know

6) Which of the following complex numbers is not equal to the others?

(a) $\frac{13}{2+3\text{i}}$

(b) $\frac{13}{2-3\text{i}}$

(c) $2 - 3\text{i}$

(d) $\frac{3+2\text{i}}{\text{i}}$

(e) I don't know

7) $z = \frac{3+4\text{i}}{2-3\text{i}}$. The complex number w which satisfies the equation $zw = 1$ is

(a) $w = \frac{6+17\text{i}}{25}$

(b) $w = \frac{-6-17\text{i}}{25}$

(c) $w = \frac{-6-17\text{i}}{5}$

(d) $w = \frac{6+17\text{i}}{5}$

(e) I don't know

8) The solution of the equation

$$(3 - \text{i})(z + 4 - 2\text{i}) = 10 + 20\text{i}$$

is

(a) $z = 46 - 52\text{i}$

(b) $z = 1 - 3\text{i}$

(c) $z = -3 + 9\text{i}$

(d) $z = 5 + 5\text{i}$

(e) I don't know

9) Which of the following is NOT true?

(a) $\frac{1}{\text{i}} + \text{i}^3 = 0$

(b) $\frac{1}{\text{i}^3} - \text{i} = 0$

(c) $\text{i}^4 = 1$

(d) $\frac{1}{\text{i}^2} = \text{i}^2$

(e) I don't know

10) Which of the following statements is NOT true?

(a) $z - z^*$ is pure imaginary

(b) $z + z^*$ is real

(c) zz^* is real

(d) $\frac{z}{z^*}$ is real

(e) I don't know

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Solutions to Multiple Choice Test

1) The correct answer is c)

$$\begin{aligned} 3+4i-(2-i) &= 3+4i-2+i \\ &= (3-2)+(4i+i) \\ &= 1+5i \end{aligned}$$

2) The correct answer is d)

$$\begin{aligned} (2+i)(3-2i) &= 6-4i+3i-2i^2 \\ &= 6-i+2 \\ &= 8-i \end{aligned}$$

3) The correct answer is d)

$$\begin{aligned} \left[(1-2i)+(1+i)\right](-3+i) &= (2-i)(-3+i) \\ &= -6+3i+2i-i^2 \\ &= -6+5i+1 \\ &= -5+5i \end{aligned}$$

4) The correct answer is a)

$$\begin{aligned} z &= \frac{-6\pm\sqrt{36-4\times1\times10}}{2} \\ &= \frac{-6\pm\sqrt{-4}}{2} \\ &= \frac{-6\pm2i}{2} \\ &= -3\pm i \end{aligned}$$

5) The correct answer is c)

$$p + qi = \frac{1}{12-5i}$$
$$= \frac{12+5i}{(12-5i)(12+5i)}$$
$$= \frac{12+5i}{144+25}$$
$$= \frac{12}{169} + \frac{5}{169}i$$
So $p = \frac{12}{169}, \quad q = \frac{5}{169}$.

6) The correct answer is b)

$$\frac{13}{2-3i} = \frac{13(2+3i)}{(2-3i)(2+3i)} = \frac{26+39i}{13} = 2+3i$$
$$\frac{13}{2+3i} = \frac{13(2-3i)}{(2+3i)(2-3i)} = \frac{26-39i}{13} = 2-3i$$
$$\frac{3+2i}{i} = \frac{(3+2i)i}{-1} = \frac{3i-2}{-1} = 2-3i$$
So $\frac{13}{2-3i}$ is not equal to the others.

7) The correct answer is b)

$$w = \frac{2-3i}{3+4i}$$
$$= \frac{(2-3i)(3-4i)}{(3+4i)(3-4i)}$$
$$= \frac{6-17i-12}{25}$$
$$= \frac{-6-17i}{25}$$

8) The correct answer is c)

$$(3-i)(z+4-2i) = 10+20i$$

$$z+4-2i = \frac{10+20i}{3-i}$$

$$= \frac{(10+20i)(3+i)}{(3-i)(3+i)}$$

$$= \frac{30+70i-20}{10}$$

$$= 1+7i$$

$$z = 1+7i-4+2i$$

$$= -3+9i$$

9) The correct answer is a)

$$i^4 = (i^2)^2 = (-1)^2 = 1$$

$$\frac{1}{i^3} - i = \frac{i}{i^4} - i = \frac{i}{1} - i = 0$$

$$\frac{1}{i^2} = \frac{i^2}{i^4} = \frac{i^2}{1} = i^2$$

$$\frac{1}{i} + i^3 = \frac{i^3}{i^4} + i^3 = \frac{i^3}{1} + i^3 = 2i^3 = -2i$$

10) The correct answer is d)

Let $z = x + iy$, so $z^* = x - iy$

$z + z^* = x + iy + x - iy = 2x$ so $z + z^*$ is real

$z - z^* = x + iy - (x - iy) = 2iy$ so $z - z^*$ is pure imaginary

$zz^* = (x + iy)(x - iy) = x^2 + y^2$ so zz^* is real

$\frac{z}{z^*} = \frac{z^2}{zz^*}$ zz^* is real but z^2 is not, so $\frac{z}{z^*}$ is complex.