Turn, turn, turn, turn
Teacher Notes

Introduction

As teachers we know that when you repeatedly rotate an object by an angle of 90°, after four rotations the final image comes back to lie on the original object—***if*** the same centre of rotation is used each time. But is that true ***if and only if*** the same centre of rotation is used? In other words is it possible for the four rotations to have different centres and the final image still to be coincident with the original object? If it is possible, then how are the four centres of rotation related? TI-Nspire provides an ideal tool to carry out this investigation and, if you have time, you may care to give it a go without further recourse to these notes!

The TI-Nspire document FourTurns.tns provides a guided investigation into the effect of four rotations of 90°. For younger students who are meeting mathematical rotation for the first time, it may be enough to consider only rotation about a single point. This can be done using just the first two problems of the document. Alternatively you may prefer students to start with a blank document and experience the delight of producing the rotations themselves.

Resources

The TI-Nspire document FourTurns.tns.

Also pages 5 and 6 of this document can be copied to provide students with full detailed instructions and key presses if you wish them to produce rotations with a new blank document.

Technical Skills required

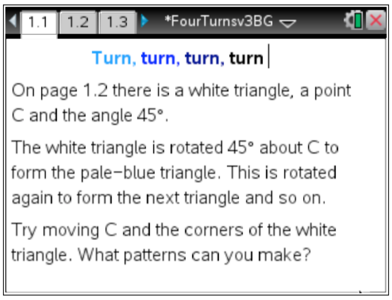
Opening a TI-Nspire document and moving from page to page.
Grabbing and dragging objects on the screen.
Simple editing of text on screen.
A little experience of using menus on Geometry pages.

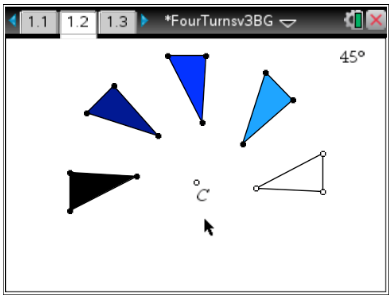
Notes about FourTurns.tns

Problem 1

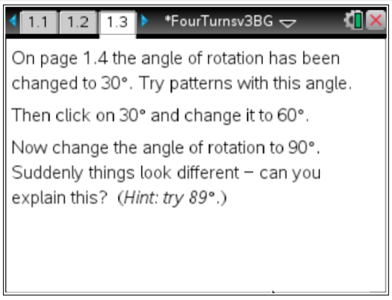
This first problem allows students to explore the effect of multiple rotations about a single point, C. It also establishes some visual conventions: points marked with open circles are draggable, the object is white and images are increasingly darker shades.

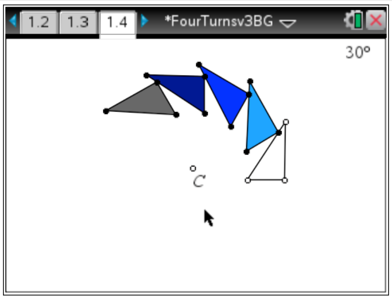
It is important to let students "play" on page 1.2: moving point C, the entire white triangle, and its vertices produces some fascinating dynamic effects and the resulting patterns may be well worth sharing.



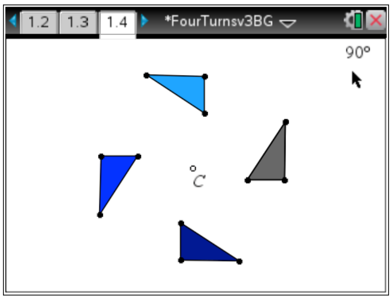


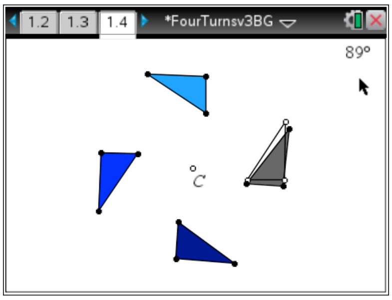
To change the angle of rotation on page 1.4, move the cursor over the text and press enter twice. Move left with ◂, delete using del , enter new numbers and press enter.





When the angle of rotation is 90° the white object triangle disappears underneath the black 4th image. Changing the angle to 88° or 89° will help to convince doubters that the object is still there!

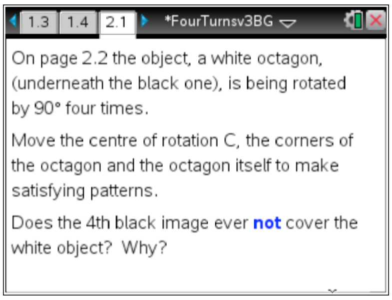


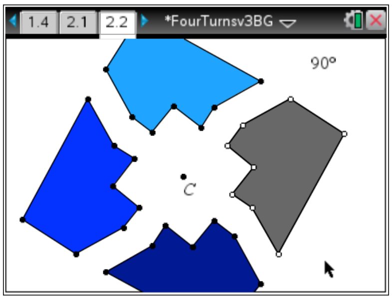


Problem 2

The aim here is to reinforce the fact that four turns of 90° about a single centre of rotation will *always* turn the object back onto itself.

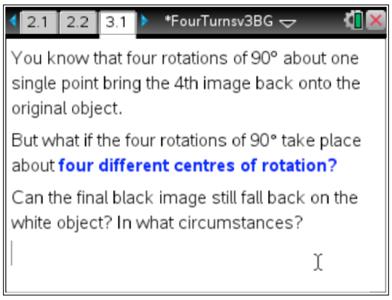
Again students will enjoy making weird and wonderful patterns!

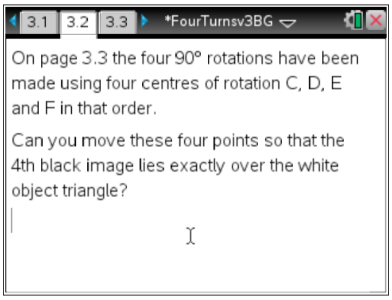




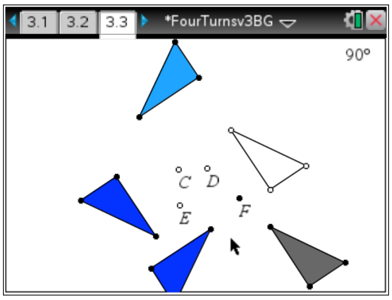
Problem 3

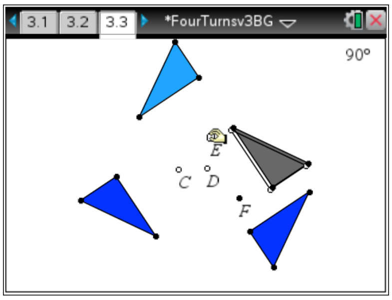
The investigation moves on to consider rotations about four different centres.



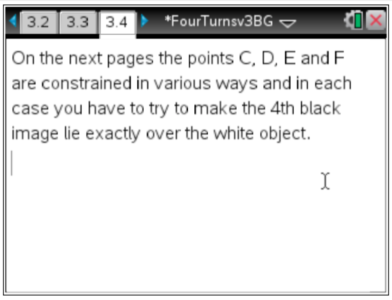


It is certainly possible to make the 4th image fall over the object, but it is far from obvious how the positions of the four centres of rotation are related.



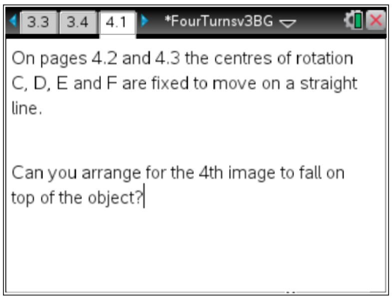


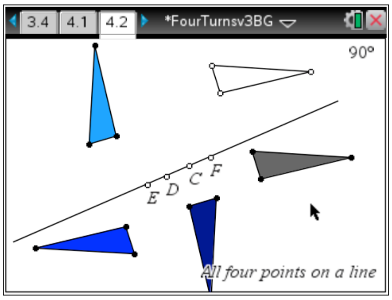
This page makes it clear that a guided investigation lies ahead. If you wish students to investigate freely without help they should not go any further through this file!



Problem 4

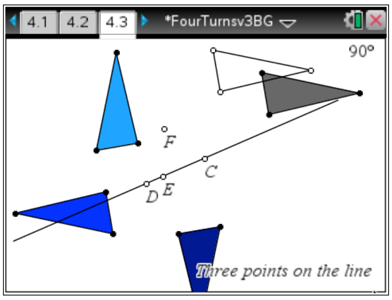
Four points on a line is tricky, although an obvious solution is to make all four centres coincident. The only other possibility is to place C and E together and D and F together.

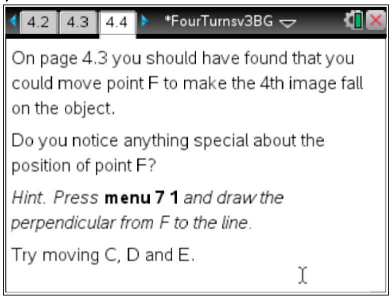




With point F free to move more solutions are possible.

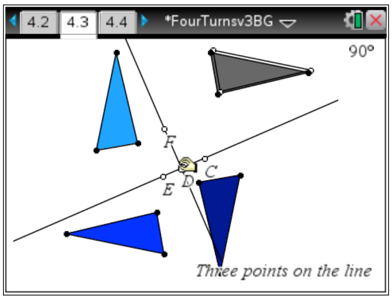
Once students have chosen the Perpendicular tool they simply need to select point F and then the line through C, D and E.





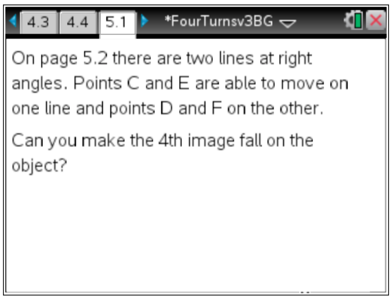
Drawing a perpendicular from point F to the line on which C, D, and E move should make it clear that, when a solution occurs, the perpendicular meets the line at D.

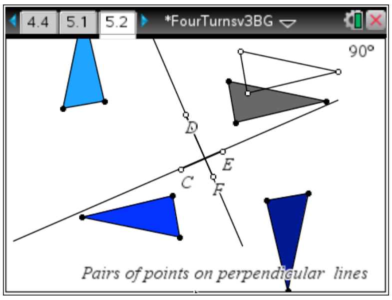
Crucially, DF is perpendicular to CE, which leads to trying a situation where D can also move along the perpendicular line. This is tested in the next problem.



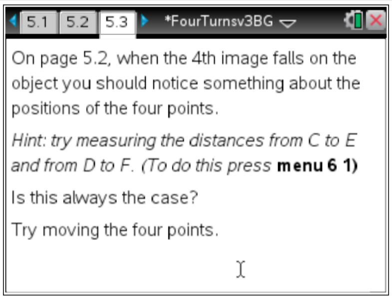
Problem 5

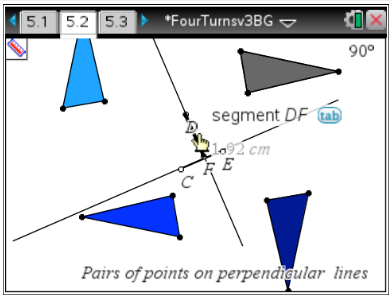
Again, solutions are possible and it should become clear that when they occur the length of segment DF equals the length of segment CE.





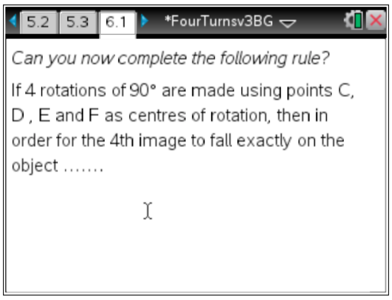
Once students have chosen the tool to measure lengths they need only to hover over the two segments to see the measurements.

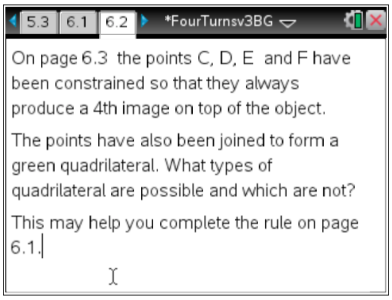




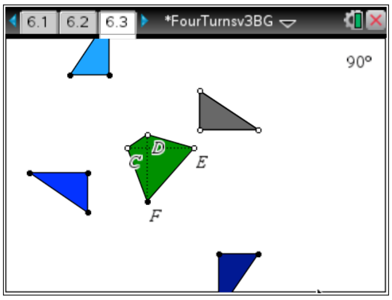
Problem 6

The rule may be completed as “the line CE must be perpendicular to the line DF and their lengths must be equal”.





When the points form a quadrilateral the rule may be completed as follows: “the quadrilateral formed by C, D, E and F must have diagonals that are perpendicular and equal”. All squares have this property and some trapezia and kites, but no rectangles or parallelograms (that are not squares) can be formed.

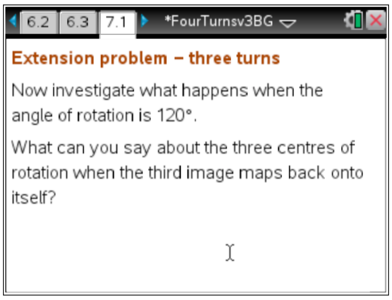


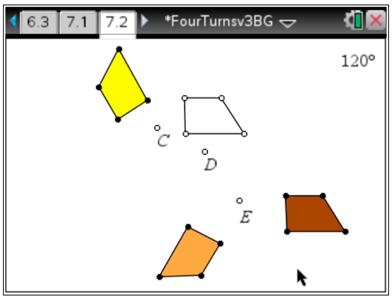
The rule is a necessary condition but not a sufficient one: not all lines CE that are perpendicular and equal in length to DF will ensure that the 4th image returns to the original object. For example, on page 5.2 if you try moving point F up past D along the line so that FD=CE the 4th image is nowhere near the object.

In terms of the quadrilateral this condition translates into a requirement that the vertices CDEF are labelled in a clockwise direction.

Problem 7

This is actually a much simpler problem, which students may well be able to solve without help. The solution has resonances with the four turns problem: it is necessary for the three points to form an equilateral triangle: again it must be labelled clockwise!



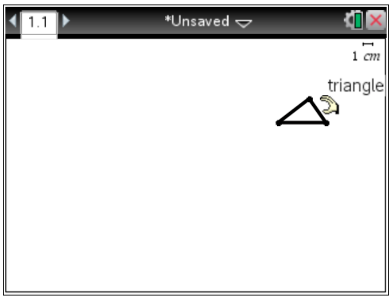


Four rotations starting from scratch

Here are the instructions that enable you to investigate four turns starting with a blank document.

Open a new document with a Geometry page
press on 1 3.

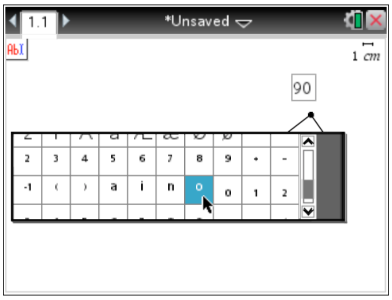
Choose to **draw a triangle** to be the object to rotate:
press menu 5 2,
move the cursor and press enter,
move to a second corner and press enter,
move to a third corner and press enter,
press esc.



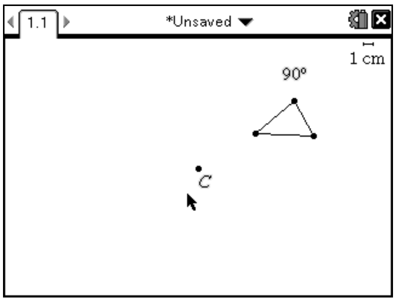
To **translate the triangle** to another position, grab and drag it:
hover over a side until you see the word *triangle* and an open hand,
press ctrl or press and hold until the hand closes,
drag the triangle to where you want it and press enter.

To **change the shape** of the triangle grab and drag a corner:
hover over a corner until you see the word *point,*
press ctrl or press and hold until the hand closes,
drag the point to where you want it and press enter.

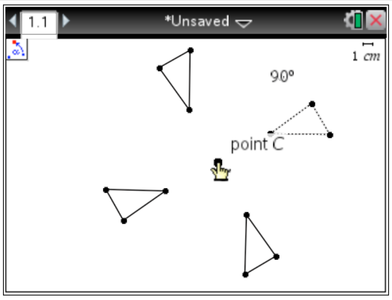
Enter the angle for the rotations, 90°:
choose Text from the Actions menu: menu 1 7,
move the cursor to a suitable position and press enter,
press 9 0 ctrl and move down to the ° symbol,
press enter enter.



Mark a point for the centre of rotation, C:
choose Point by pressing menu 4 1,
move to where you want the centre of rotation,
press enter shift C esc.



Rotate the triangle through 90° about point C:
select Rotation from the Transformations menu by pressing menu 8 4,
move to the Triangle, Point C, and the angle 90° and press enter after each one.
press enter once more.

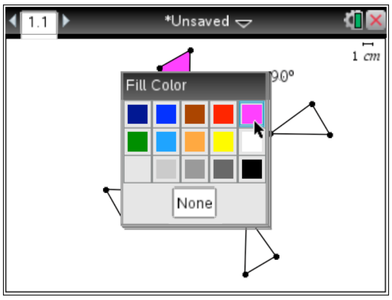


You have drawn the image of the original triangle.

Now rotate the image of the triangle, again using Point C and the angle 90° to produce a third triangle.

Repeat this twice more, rotating the previous image each time to get a new image. Where's the last image? Are you sure?

To **shade a shape:**



Hold [ctrl] and click on the shape to see the contextual menu. Press [B] [2] to choose Fill Color and click on the colour you want.

Now you can see what happens when you grab and drag point C the centre of the rotation. The original triangle or its corners can also be moved. (Press [tab] before you try to grab it.) Try to make the fourth image appear anywhere else than on top of the original object.

