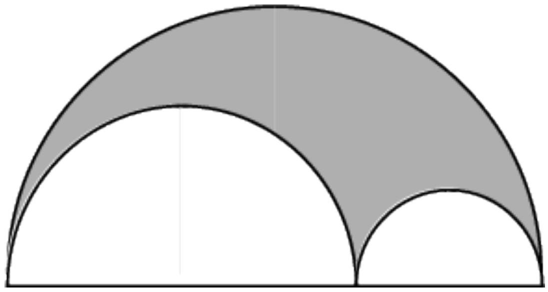
Arbelos
Teacher Notes

Introduction



The **arbelos** is a composite shape made from three semicircles defined such that the sum of the diameters of the two smaller semicircles is equal to the diameter of the largest semicircle. The point between the two smaller semicircles can be located anywhere on the diameter of the largest semicircle, and so there are many different arbeloi for each largest semicircle.

Archimedes (circa 250BC) is the first mathematician known to have studied the properties of this shape. He named it an **arbelos**, the Greek word for a shoemaker's knife.

There is a relationship between the perimeter of the arbelos and the radii of the semicircles; also between the area of the arbelos and the radii of the semicircles. The purpose of this activity, which has been developed from an old Scottish Standard Grade Investigation, is to investigate these relationships and conjecture a formal relationship between them.

Resources

Students will need a copy of the TI-Nspire document entitled Arbelosv2 transferred onto their handhelds.

Length of time required (approx)

The time required to undertake the core activity using the TI-Nspire file would be approximately 1-1½ hours. The practical pre-activity takes approximately ½ an hour.

Skills required

To undertake this activity, students will need to be able to:

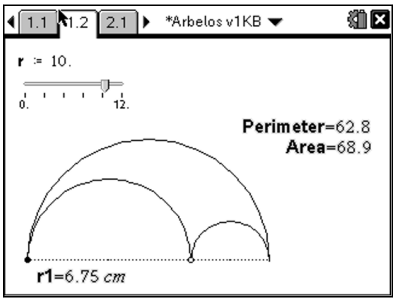
- open a TI-Nspire document and move between the pages of the document;
- move the cursor around the screen;
- increase/decrease a variable using a horizontal slider;
- grab, move and release points, lines and curves;
- gather data manually using ctrl .;
- clear a set of data by selecting the shaded box in a column of data and pressing enter twice;
- enter a function into the graph entry line;
- use the WINDOW menu to quickly resize the window (choose either 9:Zoom-Data or A:Zoom-Fit)

The activity

Prior to undertaking the TI-Nspire activity, it might be useful to have pupils create their own accurate drawing of an arbelos and to calculate its perimeter and area by hand. They can then group their arbeloi according to the diameter of the largest semicircle and compare their results for perimeter and area.

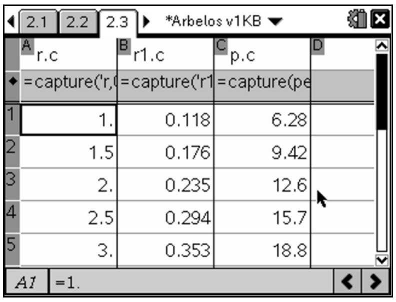
This familiarises them with the shape, reinforces their knowledge of formulae for circumference and area of a circle (and a semicircle), starts them thinking about the similarities and differences of their answers and provides a set of colourful pictures for display.

Problem 1 sets the scene for the investigation. On page 1.2, the radius of the largest semicircle can be changed using the horizontal slider with variable **r** (to integer values between 0 and 12). The point between the two smaller semicircles (denoted by an open circle) can be grabbed and moved along the diameter of the largest semicircle. The variable **r1** gives the radius of the left-hand smaller semicircle.

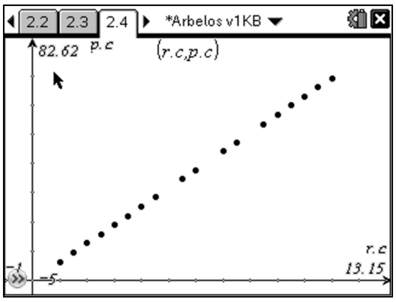


As the variables **r** and **r1** are changed, the perimeter and area of the arbelos are displayed on the page.

Problem 2 investigates the relationship between the perimeter of the arbelos and the radii of the semicircles. Using page 2.2, once pupils realise that the perimeter is affected only by variable **r**, they are then asked to gather data manually for several options for **r**. Once gathered, the data is automatically stored in page 2.3.

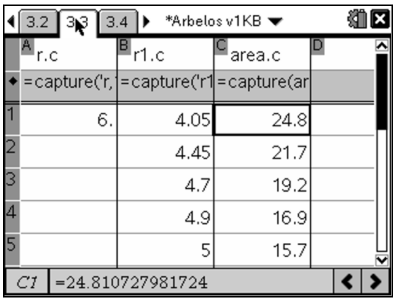


A scatter plot of perimeter (**p.c**) against largest radius (**r.c**) is automatically generated on page 2.4. Hopefully pupils will recognise that it is a linear relationship. They can then plot the function $f1(x) = x$ onto the page (use ctrl G to view the graph entry line) and grab and twist the end of the line to give a line of best fit.



If desired, they can also calculate the correct gradient of the line by defining another column in page 2.3 as perimeter divided by radius (**p.c/r.c**).

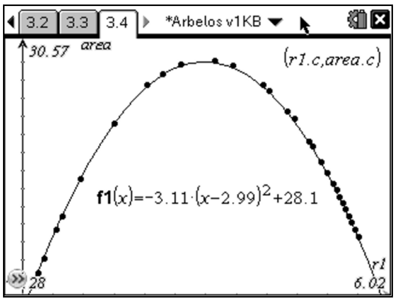
Pupils should discover that the relationship is $P = 2\pi r$, regardless of the location of the point between the two semicircles.



Problem 3 investigates the relationship between the area of the arbelos and the radii of the semicircles.

This time, data is automatically gathered into page 3.3 and a scatter plot generated in page 3.4. If pupils change both variables (**r** and **r1**), they will see that the scatter plot is a random scatter. If pupils change only **r**, then the two smaller semicircles remain in proportion to each other and **r1/r** does not change.

Pupils should choose a value for **r** in page 3.2, clear the automatic data capture values in page 3.3, then move back to page 3.2 and change the **r1** values.



Hopefully pupils will recognise the scatter plot in page 3.4 as a negative quadratic curve. They can then plot the function $f1(x) = -x^2$ onto the page (use ctrl G to view the graph entry line), move the turning point of the curve to the turning point of the scatter plot, then grab and twist the end of the curve to give the best fit (pupils may need to resize the window several times).

This is a much more complicated relationship to conjecture, especially since the function is given in completed square form ($y = a(x-b)^2 + c$). They should recognise relatively easily that $a = -\pi$ and that $b = \frac{1}{2}r$. It is much harder to recognise that $c = \frac{\pi}{4}r^2$ (a useful algebraic exercise!).

In standard quadratic form, the relationship is
$A = -\pi r_1^2 + \pi r r_1$.

A possible extension exercise is to ask pupils to prove algebraically the relationships they have discovered.